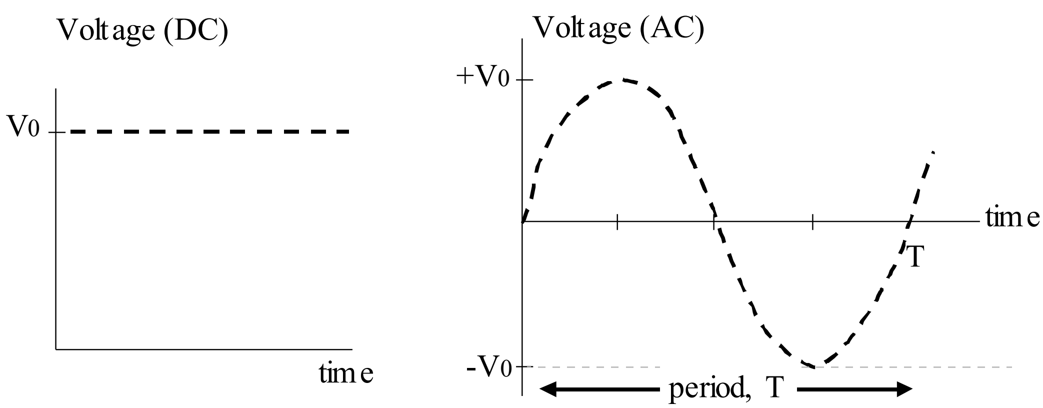
We've been cheating a bit up until now, because the 120 V in your wall socket isn't quite like a battery: V really oscillates in time:

**AC Voltage and Current:**

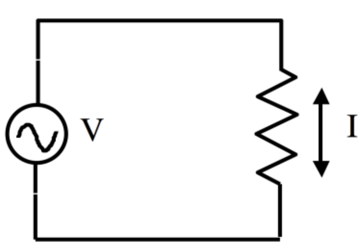
Batteries produce a steady, fixed voltage, called DC, or direct current. (We should probably call them DV, direct voltage, but never mind)

The power company produces a time-varying voltage, AC, or *alternating current.* Here's a sketch of voltage vs. time:




The mathematical formula for AC voltage is
$V(t) = V_0 \sin(2 \text{ pi } f\, t)$. (Your calculator MUST be in radian mode!)
$V_0$ is called "peak" or "maximum" voltage.
In the USA the period T = 1/60 s, so frequency f = 1/T = 60 Hz.
(In Europe, it's closer to 50 Hz).

A simple circuit diagram for a light bulb (which is basically a resistor) plugged into the wall might look like this:




The squiggle on the left is the symbol for an AC voltage source, rather than a battery.

The current no longer has a definite direction: since the voltage changes with time, so does I.

In fact, we can figure out I(t) easily from Ohm's law:

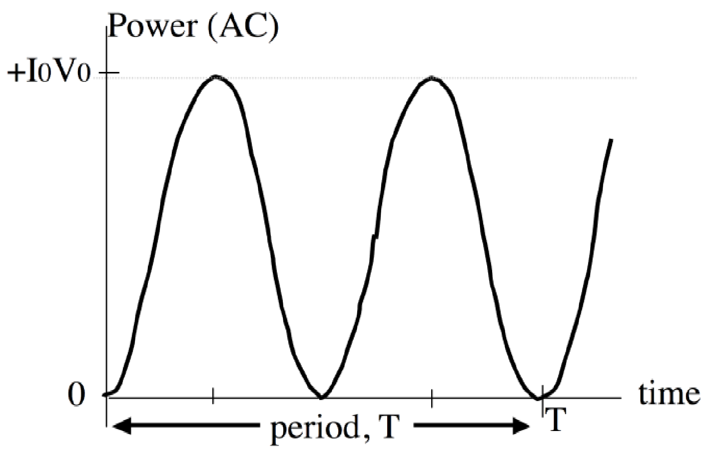
$$I(t) = V(t)/R = \frac{V_0}{R}\sin(2\pi f t) = I_0 \sin(2\pi f t).$$

Here, we have found the maximum current $I_0 = V_0/R$.

Clearly, current I alternates right along with V (hence the name AC)
Ohm's law continues to hold in AC circuits, and $V_0 = I_0 R$...
The voltage V(t) is symmetric, it's + as often as -, it averages to 0.
We say V(ave)= 0 (or $\bar{V}=0$)
Similarly, I(t) is also oscillating about 0, I(ave) = 0.

Recall that P = IV, so what is the average power, $\bar{P}$?
(You *might* guess zero, but think about light bulbs: average power used by real bulbs surely can't be zero, otherwise they'd be free)

At any moment in time, P(t) = I(t)*V(t) = $I_0 V_0 \sin^2(2\pi f t)$.
Let's graph this, because the "sin squared" changes things a bit:




sin^2(anything) is always positive.
sin^2(anything) runs from 0 up to 1 and back again.
On average, it is 1/2.
That means $\bar{P} = (1/2) I_0 V_0$. (It is *not* zero.)

Here's an odd question: "what's the average of the *square* of voltage?" (It may not seem obvious why I'd care, but then remember P = V^2/R, so V^2 *does* appear in formulas... We really will care about this.)

Remember, V(ave)=0. So you might think the average of V^2 is zero - but no! Just like the power example above: $V^2(t) = V_0^2 \sin^2(2\pi f t)$, and the average value of sin^2 is 1/2, not 0: $\boxed{V^2(\text{average}) = V_0^2/2}$.
The "average of the square" is NOT the square of the average (which was zero)!

Now we have another way to find average power: find the average of P=V^2/R, which (we just showed) is $P(\text{ave}) = (1/2)V_0^2/R$.

(It's really the SAME result as the previous page, namely
$P(ave) = (1/2)\, I_0 V_0$, because remember $I_0 = V_0/R$.)

People have even given a name to Sqrt[V^2(average)].
They call this *Vrms, the "root mean square" voltage.*

$$V_{RMS} \equiv \sqrt{(V^2)\text{Average}}$$

From its definition (just square both sides): Vrms^2 = V^2(ave).
Now, plugging in my result above for V^2(ave) = (1/2) $V_0$^2 gives

Vrms = $V_0$ / Sqrt[2] .

The rms voltage is *not* the average voltage (which is 0), but it's kind of a "representative" voltage. After all, voltage runs from -$V_0$ to +$V_0$, it's (almost) always less then $V_0$, so $V_0$/Sqrt[2] is kind of a more "typical" voltage...

Similarly, Irms = $I_0$/ Sqrt[2] gives the typical current.

Now, remember, we had old (DC) formulas that said
P = IV = I^2 R = V^2/R.
The new (AC, but averaged) formulas we just derived say
P(ave) = (1/2) $I_0$*$V_0$ = (1/2) $I_0$^2 = (1/2) $V_0$^2/R.

That recurring factor of (1/2) is annoying, and perhaps confusing.
It's there because we're writing average power in terms of *maximum* I and V.
If instead we rewrote P(ave) in terms of rms values, we'd get a nicer result:

P(ave) = Irms* Vrms. (Check that - convince yourself it's right.)

The AC average formula LOOKS like the old DC formula, exactly:
no factors of 2 at all, if you just use rms values instead of "peak" values.

P(ave) = $I_{rms}$*$V_{rms}$ = $I_{rms}$^2 R = $V_{rms}$^2/R

(Convince yourself that they're *all* right)

*Example:* US Wall sockets really have Vrms = 120 V. What is the *peak* voltage, V0?

*Answer:* $V_0$ = Sqrt[2]*Vrms = 170 V.
The US wall voltage is NOT running from -120 V to +120 V.
It's really running from -170 V to +170 V.
On average it is 0, but it has a "typical" value of 120 V.

In particular, when computing POWER, you can just pretend it's 120 V DC, and just use the old familiar power formulas.
That's why people say "the wall is 120 V"; they really MEAN Vrms.

In Europe, Vrms =240 V. This causes serious problems if you try to plug a US appliance into a European socket, or vice versa.

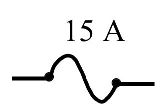
E.g., consider a 100 W bulb purchased in the US. Plug it into the wall in Europe. The resistor, R, is the same of course, but V is different.
Since P(ave) = Vrms^2/R, and Vrms is about 2 times bigger there, squaring gives 4 times more power. It becomes a 400 W bulb, but it's not designed to dissipate all that heat - it'll burn out immediately.

(If you go the other way, and plug a European 100W bulb into the wall here, what will happen?)

*Example:* Earlier, we computed R for a short piece of Cu wire, and got 0.01Ω. What's the average power dissipated in this wire, if a current of Irms = 25 A runs through the wire into a big appliance?

*Answer:* P(ave) = Irms^2*R = (25A)^2*(0.01Ω = 6.3 W.
That's quite a lot of power.(A toaster might dissipate 500 W or so) The wire will certainly heat up. Even though R is small, it's not 0. If you let appliances consume too much current, the wires in your house could easily start a fire. That's why you have *fuses* (typically rated at 15-20 A). A fuse is a little device that shuts off all the current in a circuit if it ever exceeds the rated value.

The circuit symbol for a 15 A fuse is



**Electric costs:**

The power company does not charge for power (!)
It sells you *energy*. Power = Energy/time, so
Energy = Power*time.

They don't really care if you use a LOT of power for a short time,
or a little power all the time, they charge you for the product: energy.

My public service bill charges me about 10 cents/ (kW hr).
That's pretty cheap by national standards, by the way, and I'm paying 2 cents more than most people to get wind-power...

Those are really the units on my bill. A kW·hr is a unit of ENERGY,
1 kW·hr = 1000 W * 1 hr = 1000 J/s * (3600 sec/hr) = 3.6E6 J.
(10 cents for 4 Mega Joules of energy. It's really stunningly cheap!!)

*Example:* You buy an electric space heater, rated at 1 kW.
(That's a pretty typical power rating for a big appliance. A blow-dryer might even be 2 kW)
You leave it on in the basement, day and night, from October - March, through the cold months.
How much have you paid, extra, on your electric bills?

*Answer:* Let's convert 6 months into hours:
6 mo * (30 days/month) * (24 hrs/day) = 4300 hours.

You used energy E = P(ave)*time = 1 kW * 4300 hrs = 4300 kW·hrs.

Cost to you is about 4300 kW·hrs * 10 cents/kW·hr = $430. Yikes!

(There are more efficient ways of heating a space than electric heaters. All-electric heat in houses is pricey and it'll only get worse as energy costs start climbing in the near future! Might be better to at least first insulate the room well.)